Nodal Period of an Earth Satellite

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IN an earlier paper[1] on the effect of the earth's oblateness on the period of a satellite, the present writer derived an expression for the nodal period for nearly-circular orbits. Unfortunately, the validity of this work was blighted by the assumption of constant radius in the integration. Because of recent references[2, 3] to this paper, it is in order to point out that, as part of a general analysis of oblateness perturbations, an expression for the nodal period has been derived which is valid for arbitrary orbit eccentricity.

Specifically, in terms of *osculating* elements at any ascending node, the time interval to the succeeding ascending node is

$$T_N = \frac{2\pi a^{3/2}}{\mu^{1/2}}\left[1 - \frac{3J_2(4 - 5\sin^2 i)}{4(a/R)^2(1 - e^2)^{1/2}(1 + e\cos\omega)^2} - \frac{3J_2(1 + e\cos\omega)^3}{2(a/R)^2(1 - e^2)^3}\right] \quad (1)$$

where μ is the product of the Newtonian constant of gravitation by the mass of the earth, R is the mean equatorial radius of the earth, and J_2 is the coefficient of the second zonal potential harmonic.

The important features of this equation are that it is unrestricted with respect to eccentricity and exhibits both constant and periodic components. Note that T_N increases monotonically with orbit inclination, and, for $i \leq i_c = \arcsin(\frac{4}{5})^{1/2} = 63^\circ\!.45$ ("critical" inclination), T_N is always less than the Kepler period $2\pi(a^3/\mu)^{1/2}$. For near-polar orbits, however, it is possible for the nodal period to be greater than the Kepler period when $e > 0.14$.

To test the accuracy of our result, comparison was made between periods calculated on the basis of the foregoing equation and those derived from orbits numerically integrated on an IBM 7090. For this calculation, the constants were $J_2 = 1.08230 \times 10^{-3}$ and $\mu = 5.5303934 \times 10^{-3}$ (earth-radii)^3 min)$^{-2}$. The results for a wide variety of initial conditions are presented in Table 1. Since not all the runs were initiated at an ascending node, interpolation had to be made for the precise values of the elements at the node, which explains the deviations of a, e, i, and ω from whole numbers. The Kepler period in column 5 is merely the unperturbed period $(2\pi a^3/\mu)^{1/2}$. In column 6 are the periods calculated by means of Eq. (1), and in column 7 are the periods determined from the machine-integrated orbit. The differences ΔT are given in column 8. We observe that the ΔT are essentially random and extremely small, indicating excellent agreement between theory and numerical integration. Two additional machine runs were made in which the J_4 potential term was included, but in neither case was there any change in T_N to the accuracy given in the table.

In those terms in (1) with J_2 as coefficient, it is immaterial whether one uses osculating elements or mean values, for periodic variations in a, e, i, ω are of order J_2, and any differences in T_N will be of order J_2^2. On the other hand, in the leading term it is essential to use the osculating value of a at the node.

The long-period fluctuations in semimajor axis may be eliminated to reveal more clearly the periodic variations in nodal period. Utilizing the appropriate relations from Kozai[5] or Ref. 4, we obtain

$$T_N = \frac{2\pi a^{3/2}}{\mu^{1/2}}\left\{1 - \frac{3J_2}{4(a/R)^2(1 - e^2)^{1/2}} \times \left[\frac{(2 + 3e^2)}{(1 - e^2)^{5/2}} + \frac{(4 - 5\sin^2 i)}{(1 + e\cos\omega)^2}\right]\right\} \quad (2)$$

where now a, e, i, and ω are *mean* elements.

Kalil and Martikan[3] have examined various orbit theories from which T_N may be derived and offer the following expression as valid for orbits of small eccentricity, $e = 0(J_2)$:

$$T_N = \frac{2\pi r^{3/2}}{\mu^{1/2}}\left[1 - \frac{3J_2(6 - 7\sin^2 i)}{8(r/R)^2}\right] \quad (3)$$

where r is the "average orbital radius." Direct comparison with (1) or (2), even for $e \ll 1$, is difficult because of the uncertainty as to how to relate a to r. Only in the limiting case of the zero inclination circular orbit can unambiguous comparison be made. If one substitutes the osculating semimajor axis for this orbit into Eq. (1), one finds exact agreement with Eq. (3).

Sturms[2] expresses his result for T_N, again for small eccentricity, in terms of osculating elements at some epoch. If the epoch is taken at the ascending node, a direct comparison can be made with our results. In this case his Eq. (22) becomes

$$T_N = \frac{2\pi a^{3/2}}{\mu^{1/2}}\left[1 - \frac{3J_2(6 - 5\sin^2 i)}{4(a/R)^2}\right] \quad (4)$$

in agreement with the Eq. (1) for $e = 0(J_2)$.

References

[1] Blitzer, L., "Effect of earth's oblateness on the period of a satellite," Jet Propulsion **27**, 405–406 (1957).

Table 1 Nodal periods from Eq. (1) and from numerical integration

(a/R)	e	i	ω	T_k, sec, Kepler	T_N, sec, Eq. (1)	T_N, sec, IBM 7090	ΔT
1.2	0.10	60°	0°	6,663.84	6,652.76	6,652.79	−0.03
1.2	0.10	90°	0	6,663.84	6,656.66	6,656.69	−0.03
1.20007	0.100261	10.0007	45.1130	6,664.42	6,642.24	6,642.20	+0.04
1.20017	0.100299	45.0022	45.0754	6,665.21	6,650.76	6,650.75	+0.01
1.20026	0.100339	90	45.0355	6,666.05	6,659.83	6,659.85	−0.02
1.20010	0.100215	45.0012	89.9639	6,664.71	6,651.31	6,651.30	+0.01
1.19944	0.098448	44.9956	180.1567	6,659.21	6,646.57	6,646.56	+0.01
2.0	0.00	45	0	14,338.31	14.328.13	14,328.13	0.00
2.0	0.05	45	0	14,338.31	14.327.56	14.327.56	0.00
2.0	0.10	45	0	14,338.31	14,326.70	14,326.70	0.00
2.0	0.50	45	0	14,338.31	14,289.52	14,289.62	−0.10
2.00004	0.050062	45.0003	45.0426	14,338.69	14,328.11	14,328.11	0.00
2.00010	0.100107	45.0008	45.0272	14,339.36	14,328.18	14,328.17	+0.01
2.00180	0.500540	45.0051	45.0183	14,357.69	14,320.65	14,320.77	−0.12

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[2] Sturms, F. M., Jr., "Nodal period for a circular earth satellite," ARS J. **32**, 1037–1039 (1962).

[3] Kalil, F. and Martikan, F., "Derivation of nodal period of an earth satellite and comparisons of several first-order secular oblateness results," AIAA J. **1**, 2041–2046 (1963).

[4] Blitzer, L., "The orbit of a satellite in the gravitational field of the earth," Space Technology Lab. Rept. 8655-6020-RU000 (1963).

[5] Kozai, Y., "The motion of a close earth satellite," Astron. J. **64**, 367–377 (1959).

[6] Merson, R. H., "The motion of a satellite in an axi-symmetric gravitational field," Geophys. J. **4**, 17–52 (1961).

[7] Claus, A. J. and Lubowe, A. G., "A high accuracy perturbation method with direct application to communication satellite orbit prediction," Astronaut. Acta (to be published).

Factors Influencing Electrically Sprayed Liquids

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Electrical spraying was accomplished in vacuo by allowing a liquid to emerge from a metal capillary tube maintained at a high positive potential with respect to a grounded electrode located 1 to 4 cm in front of the capillary tip. Instability and subsequent dispersion of the liquid meniscus into numerous charged particles took place at the tip of the capillary tube. The vacuum surrounding the spraying region was maintained at about 5×10^{-6} torr.

Current distributions of specific charge (charge-to-mass ratio) produced by various liquids were measured under steady-state spraying conditions with a quadrupole mass spectrometer and under naturally pulsed conditions with a time-of-flight mass spectrometer. Only those particles that were sprayed on or near the axis of symmetry of the spectrometer were intercepted and analyzed. Pure liquids and solutions containing suspensions and dissolved ionic conductors were sprayed electrically. The electrical conductivity of the various liquids ranged approximately between 10^{-4} and 10^{-15} mho/cm. No liquid was found which could not be electrically dispersed.

A summary of some of the measurements is presented in Table 1. The specific charge corresponding to the peak of a distribution is called the average specific charge. In general, it was found that the average specific charge produced by the pure liquids was less than for the liquid solutions. Silicone oil, possessing the smallest conductivity of all liquids tested, was the only liquid whose dispersed particles could not be detected. This was probably caused by the emitted particles having specific charges too low to be resolved by the spectrometer, i.e., below 0.001 coul/kg.

High values of specific charge were obtained from a mixture of 2% by weight of Darco grade S-51 activated carbon particles in glycerine and from a mixture of 2% Cab-O-Sil H-5 (a paint thickener composed of silicon dioxide particles) in glycerine. It was also found that under certain conditions the average specific charge produced by dibutyl phthalate corresponded to singly charged double molecules (diamers) or about 10^5 coul/kg.

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Table 1 Specific charge measurements for various liquids

Liquid	Pressure, cm of liq.	Voltage, kv	(q/m) avg, coul/kg
2% Cab-O-Sil in glycerine	16	17.21	300
2% Carbon in glycerine	30	17	200
Glycerine	17	17	19
	64	4.1	0.42
	58.5	3.8	0.125
2% KOH in glycerine	63.5	3.5	6.74
	1 atm	3.5	3.38
Bu_4NPi in octoil	60	9	1.67
	120	9	0.71
	10	2.1	0.05
Narcoil-40	62	3.3	0.33
	62	3	0.19
	62	2.1	0.05
Tri cresyl phosphate	65	2.2	0.021
Octoil-S	300	9	0.20
	10	4	0.12
	1 atm	4	0.03
Tetraethylene glycol	60	3	0.20
	1 atm	3	0.097

The light from a microscope illuminator and an arc lamp was focused at different times onto the emitting region of the capillary tube. This apparently caused sufficient heating of the liquid at the capillary tip to influence the specific-charge distribution. It was found that, as the temperature was increased in steps from 25° to 56°C, the average specific charge for glycerine increased from 0.4 to 3 coul/kg, and the distribution narrowed. This result was not altogether expected. It is possible that photoeffects are present in addition to temperature effects. The temperature was measured with a thermocouple.

In general, the particular spraying mode changed with increasing applied voltage. At voltages near the minimum spraying potential, a single jet of sprayed particles was usually formed along the axis of symmetry. Throughout a higher voltage range, several jets were often observed to spray from the periphery of the capillary tip and to change in number and direction with changes in voltage. Frequently the mode of spraying was such that none of the sprayed particles was intercepted by the spectrometer. Consequently, data could be obtained only for certain applied-voltage ranges. However, it was observed that the average specific charge increased as the applied voltage raised to a power up to about 3.5.

All measurements indicated the average specific charge decreased with an increased hydrostatic liquid pressure or, by Poiseuille's equation, mass-flow rate. At low liquid pressures, the average specific charge tended to remain constant, but the repeatability of data at low pressures was difficult. This phenomenon was attributed to natural pulsations of the spraying mechanism caused by the low flow rate.

The current produced by the spraying process was measured by observing the voltage across a 1-meg resistor inserted in series with the ground electrode. At voltages near the minimum spraying potential, the spraying was generally pulsating in nature and featured the liquid necking down into a slender axial filament from the end of which the spraying actually took place. It is possible that a particular combination of capillary size, voltage, pressure, spacing, and other parameters will allow continuous and nonpulsating spraying. Although such a combination so far cannot be predicted theoretically, it can be obtained experimentally.

The current pulsing that occurred when glycerine was sprayed was strongly dependent upon the accelerating voltage.